Dimensionless parameters

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May 22, 2006

In a dimensionless analysis it can be shown (Connor 1977) that, if transport is dominated by plasma physics, the energy confinement may be expressed as :

$$B_0\tau_E = F(\rho^*, \beta, \nu^*, q, \epsilon, \frac{m_e}{m_i}, \frac{T_e}{T_i}, Z_{eff}, ..., \kappa, \delta, M_{rot}...)$$

where B_0 is the magnetic field on the magnetic axis, τ_E is the energyconfinement time, ρ^* is the normalized ion Larmor radius, β is the normalized plasma pressure, ν^* is the normalized collision freauency, q is the safety factor, ϵ is the inverse plasma aspect ratio, m_e and m_i are respectively the electron and the ion mass, T_e and T_i are respectivelly the electron and the ion temperature, Z_{eff} is the effective ion charge, κ is the plasma elongation, δ is the triangularity of the plasma and M_{rot} is the Mach number of the plasma rotation defined by $M_{rot} = V/C_s$ where V is the local rotation and C_s is the ion sound velocity.

For the thermal particules j, the velocity in the plane perpendicular to the magnetic field is define by :

Wesson definition:

$$v_\perp^2 = 2v_{T_j}^2$$

with

$$v_{T_j} = \left(\frac{T_j}{m_j}\right)^{1/2}$$

In this case:

$$v_\perp = \left(\frac{2T_j}{m_j}\right)^{1/2}$$

Note that the $v_\perp$ can be defined with or without a factor $\sqrt{2}$ in the following we will use definitions with this factor.

1 Definition of ν^*

Normalisation with the bounce frequency of the considered particule (it is a neoclassic approach):

$$\nu_j^* = \frac{\nu_j}{\epsilon\omega_{b_j}}$$

Generally for ν^* definition we consider the collisions between ions and electrons, in this case:

$$\boxed{\nu^* = \nu_{ie}^* = \frac{\nu_{ie}}{\epsilon\omega_{b_e}}}$$

WARNING !!!!

The electron-ion collision frequency from the Wesson and taken by Knud Thomsen is:

$$\boxed{\nu_{ie} = \frac{1}{\tau_e}}$$

But when we condider $v = v_T$, the usual definition of ν_{ie} given by Hinton Hazeltin 1976 (Rev. Mod. Phy.) is:

$$\nu_{ie} = \frac{3\sqrt{\pi}}{4\tau_e}$$

The electron collision time is (wesson):

$$\tau_e = 3(2\pi)^{3/2}\frac{\epsilon_0^2 m_e^{1/2} T_e^{3/2}}{n_i Z^2 e^4 ln\Lambda}$$

The bounce frequency is given by :

$$\omega_{b_j} = \frac{v_j \perp}{qR}\left(\frac{r}{2R}\right)^{1/2}$$

Considering the electron-ion collision frequency, the normalization is done with $\omega_{b_e}(r = a)$:

$$\omega_{b_e} = \left(\frac{T_e \epsilon}{m_e}\right)^{1/2}\frac{1}{qR}$$

For a comparaison with the other studies, we will take $\nu_{ei} = \dfrac{1}{\tau_e}$

In this case:

$$\nu_{ie}^* = \frac{\nu_{ie} Rq}{\epsilon^{3/2} v_{T_e}}$$

$$\nu_{ie}^* = \frac{n_i Z^2 e^4 ln\Lambda}{3(2\pi)^{3/2}\epsilon_0^2}\frac{qR}{T_e^2 \epsilon^{3/2}}$$

$$\textit{From temperature express in keV} : \nu_{ie}^* = \frac{n_e[m^-3] Z e^4 [c] ln\Lambda}{3(2\pi)^{3/2}\epsilon_0^2}\frac{qR[m]}{(T_e[keV] * 1000/e[c])^2 \epsilon^{3/2}}$$

2 Definition of ρ^*

$$\boxed{\rho^* = \frac{\rho_i}{a}}$$

where ρ_i is the ion Larmor radius defined by:

$$\rho_i = \frac{v_\perp}{\omega_{ci}} = \frac{m_i v_\perp}{e_i B}$$

$$\rho_i = \frac{\sqrt{2 m_i T_i}}{eB}$$

$$\boxed{\rho^* = \frac{\sqrt{2 m_i T_i}}{eaB}}$$

$$\textit{From temperature express in keV} : \rho^* = \frac{(2 m_i[kg] T_i[keV] * 1000/e[c])^{1/2}}{e[c] a[m] B[T]}$$

3 Definition of β

$$\boxed{\beta = \frac{P}{B^2/2\mu_0} = \frac{2\mu_0}{B^2}(n_eT_e + n_iT_i)}$$

To take into account the dilution, we must condsider some impurities. For example, if we consider the Carbon:

$$\beta = \frac{2\mu_0}{B^2}(n_eT_e + (n_H + n_C)T_i) = \frac{2\mu_0}{B^2}(n_eT_e + (\frac{Z_C - Z_{eff} + 1}{Z_C}n_e)T_i)$$

The normalized β_N correspond to the β normalized with the value of the limit MHD (Troyon limit) in β:

$$\beta_N = 100 * \frac{\beta}{Ip/aB}$$

From temperature express in keV *:* $\beta = \frac{2\mu_0[Hm^{-1}]}{B[T]^2}n_e[m^{-3}]T_e[keV](1 + \frac{1}{Z})$
and $\beta_N = 100 * \frac{\beta}{Ip[MA]/a[m]B[T]}$

4 Exprimental determination

4.1 Global analysis

For global analysis, we use global parameters defined and determined as following :

⋆ **Global temperatue** $\overline{T}$

The determination of a global temperature is realised using the relation:

$$\boxed{W_{th} = 3\overline{n}\overline{T}V}$$

with W_{th} *the thermal energy,* $\overline{n}$ *the average density and* V *the plasma volume.*
Generally the thermal energy is express in MJ*, and* $T[keV] = \frac{W[MJ] * 10^3}{n[m^{-3}]V[m^3]e[c]}$

⋆ **Plasma volume** V

One solution is to determine the plasma volume V from the plasma parameters using :

$$V = 2\pi^2a^2\kappa R$$

with a *the minor radius,* R *the major radius and* κ *the triangularity of the plasma.*
But it is better to use the measured volume from the magnetic equilibrium calculation.

⋆ **Cinetic pressure** P

For the mesure of the cinetic pressure, we use the relation : $\frac{W_{th}}{V} = \frac{3}{2}\langle P\rangle$

⋆ **Thermal energy** W_{th}

The thermal energy is obtain from the MHD energy which one we substract the energy of the fast ions. ($W_{th} = W_{MHD} - \frac{3}{4}W_{fperp} - \frac{1}{2}W_{fpar}$)

⋆ **Global electron density** $\overline{n}$

For the global density we take the line integrated density. (from interefometer)

⋆ **Effective charge** Z_{eff}

⋆ **Major radius** R

⋆ **Minor radius** a

⋆ **Coulomb logaritm** $ln\Lambda$

For electron-ion collision ($T_e \geq 10eV$) with $T_e(keV)$:

$$ln\Lambda = 15.2 - \frac{1}{2}ln(n_e/10^{20}) + lnT_e$$

(Note : the value of $ln\Lambda$ is generally for AUG around 16 or 17)

⇒ **Average Temperature** $\overline{T}$

$$\boxed{\overline{T}(keV) = 1.053979 * \frac{W_{th}(MJ) * \epsilon}{nel(1e20) * a^3 * \kappa}}$$

⇒ **Larmor radius** ρ_i

$$\boxed{\rho_i = 4.5693.1e-3\frac{\sqrt{(pgas * \overline{T}(keV))}}{B(T)}}$$

with $\sqrt{2}$ factor.

⇒ **Collisionality** ν_{ei}

$$\boxed{\nu_{ei} = 9174.3 * \frac{nel(1e20) * Z * ln\Lambda(=17)}{\overline{T}(keV)^{3/2}}}$$

⇒ **Electron bounce frequency** ω_{bounce_e}

$$\boxed{\omega_{bounce_e} = 1.326199.1e7 * \left[\frac{\overline{T}(keV) * \epsilon^3}{a^2 * q^2}\right]^{1/2}}$$

⇒ **Thermal beta β_{th}**

$$\boxed{\beta_{th}(\%) = 8.05352 * \frac{nel(1e20) * \overline{T}(keV)}{B^2}}$$

4.2 General remarks

⋄ **The perpendicular velocity definition used with $\sqrt{2}$ factor ($v_\perp = \left(\frac{2T_j}{m_j}\right)^{1/2}$)**

⋄ ν^* **: For the global anlaysis the bounce frequency is taken with $r = a$ and $q = q_{95}$**

⋄ β **: With or without impurities ??**