Turbulence in Fusion Plasmas

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Informal: ask questions, interrupt the speaker, complain if you don’t understand.

Try to avoid esoteric issues though.

Mine is a bad example: Organized more like a seminar than a journal club.

Please Volunteer. (send me an email at ozgur.gurcan@lpp.polytechnique.fr)

http://difdop.polytechnique.fr/jclub

Presenting a paper:

1. Choose something that will interest most people.
2. Verify with me or one of the senior researchers that it makes sense to present that paper.
3. Try to give the main points, conclusions and the reasoning
4. Avoid too much details unless there is interest.

Presenting a subject: (1-4 above +)

Be pedantic. Use the board if it makes more sense.
Two main types of fusion devices:

- **Tokamak**
- **Stellarator**

This lecture is strictly for the tokamak. Though some results may work in some cases for other devices.

- an RFP or a spheromak are similar to a tokamak in some sense but there are still important differences in the nature of turbulence and transport in different types of machines.

A Modern day tokamak has:
- a rather high $B \sim 1 - 4 \ T$, (ITER $B_0 \sim 5.3 \ T$).
- a rather high $T_i \sim 2 - 20 \ KeV$.
- a system size of a few meters with a reasonably high aspect ratio (ITER $a = 2 \ m$, $R = 6 \ m$)
- usually $D - D$ plasma with a $n_i \sim [2 - 10] \times 10^{19} m^{-3}$
- little collisions ($T \nearrow$, $\nu \searrow$).
- Serious amount of free energy to drive turbulence.
The basic role of turbulence in tokamaks

The role of turbulence

- Heat and particles confined by the magnetic field are extremely inhomogenous.
- But fusion plasma has little collisions.
  - So each particle is “stuck” to its magnetic field line.
- But plasma particles collectively generate a time evolving electromagnetic field.
- This collective field applies fluctuating forces on the particles themselves.
- The plasma particles will drift as a result of such forces $\mathbf{v}_E = \mathbf{E} \times \frac{\mathbf{b}}{B}$.
- If these drift motions are out of phase with a heat or a density fluctuation.
  - That is if the drift is “outward” where the heat is maximum and “inward” where the heat is minimum.
- There will be a net outward heat transport. (particles that go out are always “hotter”).
- This means that in order to understand how we will loose the heat in a tokamak reactor, we need to understand how the collective electromagnetic fields evolve, and how the microscopic phase between $\Phi$ and $T$ is established.
The simplest diagnostic system (see Matthews, 1994). Point measurements of electrostatic potential. There are a gazillion variants. In order to obtain spatiotemporal dynamics, usually used in multiple-tip configurations arrays Can be used only in the edge region (SOL or edge) It gives:

- electron temperature via:

\[ T_e = \frac{\Phi_{pl} - V_{fl}}{R_{ei}} \quad R_{ei} = \left| \frac{I_{es}}{I_{is}} \right| \]

- velocity, vorticity etc. via finite difference formulas (multiple probes).
- Correlations of all these quantities, heat flux, momentum flux etc. etc.
Doppler Reflectometry

- Uses back-scattered signal instead of the reflected signal (Hennequin et al., 2004).
  - thus sometimes called Doppler backscattering.
- Uses ray or beam-tracing (i.e. WKB).

Basic properties:
- Selected wavenumber
  \[ k = k_i - k_s = 2k_i \sin(\theta/2) \] (Bragg)
- wave number resolution
  \[ \Delta k = \frac{2}{w} \] (Gaussian beam)
- poor spatial resolution.

- Allows accessing the Fourier amplitudes directly.
  - Thus ideal for spectrum measurements.
- As a side effect, it gives the poloidal velocity directly from the Doppler shift.
- Can go up to high \( k \) (e.g. \( k \sim 15 - 20 \ cm^{-1} \)).
Turbulence; but of which kind?

- Measurements of fluctuating density and electrostatic potential at the edge via probes $\delta n/n \sim \%30$.
- At the core via wave-scattering (e.g. reflectometer). $\delta n/n \sim \%1$
- Theoretically $\delta n/n \sim e\Phi/T_i$
- $\delta B/B_0 \sim 10^{-4-5}$ in L-mode core

- Magnetic fluctuations associated with energetic particles, islands, large and meso-scale physics.
- MHD activity is observed during the L-H transition, ITBs (explain!) etc.
- Small scale magnetic fluctuations may play a role in electron transport.

Tore Supra (Colas L. et al., Nuclear Fusion 1998)
Turbulence; but of which kind?

- For the generic problem of L-mode ion heat transport, the turbulence is electrostatic, and driven by the ion temperature gradients (ITG).

- It is of the type we call “drift-instabilities”:
  - convective instabilities that propagate as drift-waves in the poloidal direction.

- In particular, the turbulence in a tokamak core is widely believed to be the toroidal drift-ITG mode.
Drift Waves
reduced models: Hasegawa-Mima

Rather similar to Euler equation:

\[
\frac{\partial}{\partial t} \left( 1 - \nabla^2 \right) \Phi + \frac{\partial}{\partial y} \Phi = \mathbf{\hat{z}} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi
\]

Physics contained in Hasegawa-Mima equation:

- Simplest drift-wave description which is **nonlinear**.
- Inhomogenous background density
- Fluctuating \( \tilde{E} \times B \) flow causes density fluctuations to move
- Electrostatic field follows via

\[
\frac{\tilde{n}}{n_0} \sim \frac{e \tilde{\Phi}}{T_e}
\]

Instability appears if there is a phase shift

\[
\frac{\tilde{n}_k}{n_0} \sim \frac{e \tilde{\Phi}}{T_e} (1 - i\delta_k)
\]

Gives the simplest drift-wave dispersion relation:

\[
\omega_k = \frac{k_y}{(1 + k^2)}
\]
Reduced Models: ITG, CTEM etc.

- This is overly simple for the real plasma system.
- Today we know that the *ion temperature gradient driven instability* (ITG) is the primary cause of ion heat transport in tokamaks.
- A very simplified version of the ITG instability can be described by (Lee and Diamond, 1986)

\[
\begin{align*}
\frac{\partial}{\partial t} \left( 1 - \nabla_{\perp}^2 \right) \Phi + v_D \frac{\partial}{\partial y} \Phi + \nabla v_i \times \nabla \cdot \nabla^2 \Phi &= \hat{b} \times \nabla \Phi \cdot \nabla^2 \Phi \\
\frac{\partial}{\partial t} \nabla v_i + \hat{b} \times \nabla \Phi \cdot \nabla v_i - \mu \nabla^2 v_i &= -\nabla \Phi - \nabla P \\
\frac{\partial}{\partial t} P + v_D \left( \frac{1 + \eta_i}{\tau} \right) \frac{\partial}{\partial y} \Phi + \hat{b} \times \nabla \Phi \cdot \nabla P &= -\gamma \nabla v_i
\end{align*}
\]

- More generally, a set of gyro-fluid equations can be obtained from the moment hierarchy of the gyrokinetic equations (Brizard, 1992)
- However the approach is quantitatively inadequate (i.e. in order to describe linear physics accurately one needs a 16 moment model).
The gyrokinetic equation

- Nobody writes this in a talk, but:

\[
\left[ \frac{\partial}{\partial t} + \left( v_{\parallel} \hat{b} + v_D \right) \cdot \nabla \right] \delta g + \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla \delta g = -\frac{e}{m} \frac{\partial F}{\partial \epsilon} \frac{\partial}{\partial t} \langle \Psi \rangle - \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla F
\]

where

\[
\langle \Psi_k \rangle = J_0 \left( \frac{k_{\perp} v_{\perp}}{\Omega_i} \right) \left[ \tilde{\Phi} - \frac{A_{\parallel} v_{\parallel}}{c} \right]
\]

the Bessel function comes from the average over the gyro-motion of the

\[
\langle e^{i k \cdot r} \rangle \approx \int_0^{2\pi} e^{i \frac{k_{\perp} v_{\perp}}{\Omega_i} \sin \theta} d\theta = J_0 \left( \frac{k_{\perp} v_{\perp}}{\Omega_i} \right)
\]

- Thus, the moments of the gyro-kinetic equation would give reduced 2D MHD equations.
- Apart from the fact that

\[
F = \frac{n(x)}{(2\pi T(x)/m)^{3/2}} e^{-\frac{m}{2T(x)} (v_{\parallel}^2 + v_{\perp}^2)}
\]

which is the free energy source for the instability.
The physics content in the

Gyrokinetic Equation

\[
\left( \frac{\partial}{\partial t} + v_\parallel \hat{b} \cdot \nabla + \mathbf{v}_D \cdot \nabla \right) \delta g + \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla \delta g = -\frac{e}{m} \frac{\partial F}{\partial \epsilon} \frac{\partial}{\partial t} \langle \Psi \rangle - \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla F
\]
A wonderland of linear instabilities

The gyrokinetic equation

The physics content in the

Gyrokinetic Equation

\[
\left( \frac{\partial}{\partial t} + v_\parallel \hat{b} \cdot \nabla + v_D \cdot \nabla \right) \delta g \\
+ \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla \delta g = -\frac{e}{m} \frac{\partial F}{\partial \epsilon} \frac{\partial}{\partial t} \langle \Psi \rangle - \frac{\hat{b}}{B} \times \nabla \langle \Psi \rangle \cdot \nabla F
\]

Nonlinearity

Free Energy from the Gradients
Characteristic features (ITG)

- Basic scale length is the Ion Larmor radius.
- Time scale is the diamagnetic frequency $\omega_{\star i} \propto \Omega_i \frac{\rho_i}{L_n}$. ($L_n$ is the gradient length scale)
- Thus a transport estimate (called Gyro-Bohm):
  $$\chi_i \sim \frac{\langle \Delta x^2 \rangle}{\Delta \tau} \sim \frac{\rho_i}{L_n} \Omega_i \rho_i^2$$
- We have weak wave turbulence. Linear physics is important!
- The most-unstable ITG wave-number is around $k_{\perp} \rho_i \sim 0.1 - 0.3$
- The transport is dominated by a number of wave-numbers around the most unstable mode.

The phase between different fields (i.e. $\Phi$ and $T$ etc.), remain very close to the linear phase in this range of wave-numbers.
We said the transport of heat is due to turbulence (collisions are rare etc.).

And the temperature gradient provides the free energy source for the turbulence.

So the more the temperature is confined ($\nabla T$ is large), the more turbulence there will be, and the faster we will loose the heat.

Fortunately this is not completely true. Due to the tendency of wave turbulence to drive zonal flows, which reduce turbulence, the turbulence “regulates itself”

**Why does the wave-turbulence drive zonal flows?**

In dispersive waves, it is difficult to satisfy simultaneously the Manley-Rowe relations:

$$\omega_{k_1} + \omega_{k_2} + \omega_{k_3} = 0$$

$$k_1 + k_2 + k_3 = 0$$

where:

$$\omega_k \approx \frac{k_y}{1 + k^2}$$

There is indeed a resonant manifold, but the interaction itself is weak on that manifold.

In contrast each drift mode can interact with an $\omega = 0$ mode via modulational instability (4-wave interaction).
Basic features of ITG turbulence

**Weak Turbulence**

- Which imposes a resonance-manifold $\Delta \omega = 0$:
  \[
  \frac{k_y}{1 + k^2} + \frac{p_y}{1 + p^2} + \frac{q_y}{1 + q^2} = 0
  \]

- Recall the Hasegawa-Mima system:
  \[
  \frac{\partial}{\partial t} (1 - \nabla^2) \Phi + \frac{\partial}{\partial y} \Phi = \hat{z} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi
  \]

- What is an interaction coefficient?
  \[
  \frac{\partial}{\partial t} \Phi_k + i\omega_k = \sum_{p+q=-k} M_{kpq} \Phi_p^* \Phi_q^*
  \]

- The interaction coefficient for the Hasegawa-Mima equation is
  \[
  M_{kpq} = \frac{\hat{z} \times p \cdot q (p^2 - q^2)}{1 + k^2}
  \]
Weak Turbulence

- If we include large scale flows, we can write a wave-kinetic equation (Smolyakov and Diamond, 1999; Dubrulle and Nazarenko, 1997):

\[
\frac{\partial}{\partial t} N(k, x) + \frac{\partial \omega}{\partial k} \frac{\partial}{\partial x} N(k, x) - \frac{\partial \omega}{\partial x} \frac{\partial}{\partial k} N(k, x) = 2\gamma_k N(k, x) + C(N, N)
\]

where \( N(k, x) \) is the wave-action density.

- at the same time the Reynolds stresses can drive zonal flows

\[
\frac{\partial}{\partial t} \langle \nabla^2 \Phi \rangle = \nabla \cdot \left( \hat{z} \times \nabla \tilde{\Phi} \nabla^2 \tilde{\Phi} \right)
\]

which modify the average frequency

\[
\omega(k, x) = \omega_k - \overline{v}_E k_y
\]

- Note the isomorphism with the Vlasov-Maxwell system.
- The waves live in the geography defined by the mean.
- While the mean itself is indeed generated collectively by the waves.
- Since the mean flow shear, reduces the effective growth rate, it kills its own drive → predator prey like dynamics.
The unstable modes are “ballooned” at the low field side (outside the torus).

and are elongated along the magnetic field.

As zonal flows appear the turbulence become less ballooned.

Usually the two different limits (local vs. global) are described (in analytical theory) using different formalisms.
Dynamics of the Cascade

- This means the zonal flows are important in the “cascade” process.
- They take small $k$ drift waves, and by *shearing them apart*. They send the energy down to small scales.
- They are also *minimum inertia* $(q \ll 1 \sim k)$:
  \[
  \frac{\partial}{\partial q} q^2 \Phi_q \quad \text{vs.} \quad \frac{\partial}{\partial t} (1 + k^2) \Phi_k
  \]
- So the energy deposited into ZFs is very efficient in accelerating them.
- Finally, They are *unaffected by Landau damping* ($k_\parallel = 0$ for zonal flows).
- They are mildly damped due to their interactions with trapped particles.

A simple estimate when the interactions with the zonal flow is dominant:

\[
|\tilde{\Phi}_k|^2 \sim |\tilde{n}_k|^2 \sim \frac{k^{-3}}{(1 + k^2)^2}
\]

For us:

\[
E(k) = \int k d\alpha_k \int_{k-\epsilon}^{k+\epsilon} d^2k' \left[ (1 + k \cdot k') \Phi_k \Phi_{k'} \right]
\]

\[
W(k) = (1 + k^2) E(k)
\]

- This means the above spectrum is
  \[
  W(k) \propto k^{-2}
  \]

For fluid turbulence $E(k) = k^{-2} W(k)$ so this would be $E(k) \propto k^{-4}$, $\Rightarrow$ Saffman spectrum $\Rightarrow$ sharp (potential) vorticity gradients.
I didn’t go into the analytical methods we use:

- Dimensional analysis
- Closures: EDQNM, Renormalization etc.
- Cascade Models: Shell models, DAMs etc.

I didn’t talk about the L-H transition, self organization, all the different exceptions to the basic rules I presented here.

Of course the reality is much more complex and interesting.