Effects of 3D Magnetic Perturbations on Zonal Flows in Tokamak Plasmas

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Outline

Background:

- Resonant Magnetic Perturbation (RMP) effect on LH transition
- Previous theories of RMP effects on Zonal Flows

This work:

- Tokamak ZF calculation in the presence of the 3D field
- Results and Conclusion

Ackn: H. Sugama, T. Watanabe, W.H. Ko, J.K. Park
Increase of L-H power threshold in the presence of RMP

- Recently, it has been observed in various machines that the L-H transition threshold increases when the RMP is applied.

  [P. Gohil et al., NF (2011)] DIII-D
  [S.M. Kaye et al., NF (2011)] NSTX
  [A. Kirk et al., PPCF (2011)] MAST
  [F. Ryter et al., NF (2012)] ASDEX-U
  [W.H. Ko et al., APS (2016)] KSTAR

- DIII-D team reported that the power threshold for L-H transition monotonically increases, as the relative amplitude of the radial vacuum RMP field exceeds a certain level and increases.

- KSTAR team also observed similar trend, but with $P_{th}$ change at very small RMP level.
Theories of 3D field effects on tokamak turbulent transport

- Theory of turbulence-ZF interaction
  [M. Leconte, P.H. Diamond and Y. Xu, NF 54, 013004 (2013)]

  - 2 Predator (ZF, mean flow) - 1 Prey (turbulence) model in the presence of RMP.
  - RMP induced $\delta j \times \delta B$ torques in resistive drift wave turbulence
    → enhances collisional ZF damping → turbulent transport increases.
  - 0-dimensional model.
  - Equilibrium magnetic field inhomogeneity is not considered regarding ZFs.

- Theory of magnetic flutter effect on residual zonal flows
  [P.W. Terry et al., PoP 20, 112502 (2013)]

  - Extended Rosenbluth-Hinton gyrokinetic calculation of residual zonal flows.
  - Performed a perturbative analysis of ZF response in stochastic magnetic fields.
  - Magnetic flutter $\propto \delta B_r$ induces ZF decay toward zero level.

- Neither theories have not consider effect of the parallel component of the perturbed 3D field on ZFs.
We study ZF behavior in tokamak plasmas in the presence of externally imposed 3D fields, by extending theoretical study of Sugama-Watanabe on ZF response in helical systems [Sugama-Watanabe, PRL ‘05], [Sugama-Watanabe, PoP ‘06].

The magnetic field model in our problem is as follows.

\[ B = B_{\text{tokamak}} + B_{3D} \]  
\[ B_{\text{tokamak}} \equiv B_0 = e_\theta B_\theta + e_\zeta B_0 / [1 + (r/R_0) \cos \theta], \quad B_{3D} \equiv \delta B \]  

We assume a high aspect ratio tokamak: \( r/R_0 \equiv \epsilon \ll 1 \) and \( B_\theta / B_0 \approx \epsilon / q \ll 1 \).

For the externally imposed 3D fields, we use an ordering

\[ |\delta B_\perp / B_0| \sim |\delta B_\parallel / B_0| \sim \delta \ll 1 \]  

where the small parameter \( \delta \) represents relative amplitude of the 3D field. Here we only consider toroidicity induced magnetic trapping and ignore other class of magnetic trapping (e.g., helicity induced trapping treated by Sugama-Watanabe), by assuming \( \delta \ll \epsilon \) and moderate variation of the 3D field on the poloidal and the toroidal angle.

Note that this ordering is compatible with tokamak experiments. [Y. In et al., NF ‘15]
Importance of Rosenbluth-Hinton Residual Zonal Flow

• J. Glanz [Science ‘96] reports:
  Prediction: failure of ITER (old design)
  -- based on gyro-Landau-fluid-based transport model
  -- ZF completely damped in old GF model

• Rosenbluth-Hinton [PRL ‘98]
  undamped collisionless ZF from Gyrokinetic theory

![Graph showing time and RH residual level](image)
3D field effect on ZFs in tokamaks

- The gyrokinetic Vlasov equation is used to describe ZFs ($k = S'(\psi)\mathbf{V}\psi$) in tokamaks.

$$
\left( \frac{\partial}{\partial t} + v_\parallel \mathbf{b} \cdot \mathbf{V} + i\omega_D \right) g_k = \frac{e}{T} F_0 J_0 (k_r \rho) \frac{\partial \phi_k}{\partial t} + S_k F_0.
$$

The notations for the physical quantities are typical. [Rosenbluth-Hinton, PRL ‘98]

- We consider an impulse ion ZF source $S_{ik} F_{i0} = \delta f_{ik}(0) \delta(t)$ with an initial condition

$\delta f_{ik}(0) = [\delta n_{ik}(0)/n_0] F_{i0}$.

Note that $\delta n_{ik}(0) = -\delta n_{\text{pol},ik}(0)$ from the charge-neutrality, where $\delta n_{\text{pol},i}$ is the ion classical polarization density, as we consider time scales longer than a few ion gyro-periods.

- The 0th order equation (for time scale shorter than the bounce/transit period) is

$$(v_\parallel \mathbf{b} \cdot \mathbf{V} + i\omega_{D0}) g_0 = 0,$$

where we have decomposed the magnetic drift frequency as $\omega_D = \omega_{D0} + \bar{\omega}_D$.

$\omega_{D0}$ is fast radial drift, and $\bar{\omega}_D$ is slow radial drift of bounce-center which has a secular non-zero value (for both trapped and passing particles) due to the 3D field.

- The 0th order equation yields a solution for the form $g_0 = h \exp(-iQ)$, where $\mathbf{b} \cdot \mathbf{V}h = 0$, and $Q \approx (MI/e) S'(\psi)v_\parallel/B_0$ is the orbit width.
3D field effect on ZFs in tokamaks

- The 1\textsuperscript{st} order equation is as follows.
\[ (v_{\parallel} \mathbf{b} \cdot \nabla + i \omega_{D0})g_1 = -\frac{\partial g_0}{\partial t} - i \bar{\omega}_D g_0 + \frac{e}{T} F_0 J_0(k_r \rho) \frac{\partial \phi_k}{\partial t}. \] (5)

After orbit average the equation yields
\[ \frac{\partial h}{\partial t} = \frac{e}{T} F_0 J_0(k_r \rho) \frac{\partial \phi_k}{\partial t} - i \bar{\omega}_D h, \] (6)
and its solution has an approximate expression as
\[ h(t) \approx \left[ e^{iQ \delta f_k(0)} + \frac{e}{T} F_0 e^{iQ J_0 \dot{\phi}_k(t)} \right] e^{-i \bar{\omega}_D t}, \] (7)
which is valid in both short-time limit \( t \ll \bar{\omega}_D^{-1} \) and long-time limit \( t \gg \bar{\omega}_D^{-1} \).

- Substituting the solution into the quasi-neutrality condition and following the same procedure as in [Sugama-Watanabe, PoP '06], we obtain
\[ \frac{e \phi_k(t)}{T_i} = \frac{\langle I(t) \rangle}{D(t)}, \quad \text{where } \langle \cdots \rangle \text{ is the flux-surface average and} \]
\[ \langle I(t) \rangle = \left\{ \int d^3v \ J_0 e^{-iQ} e^{iQ J_0 \delta f_{ik}(0)} e^{-i \bar{\omega}_D t} \right\}, \] (9)
\[ D(t) = \left\{ \int d^3v F_{0i} \left[ 1 - J_0 e^{-iQ} e^{iQ J_0 e^{-i \bar{\omega}_D t}} \right] \right\} + \left\{ \int d^3v F_{0e} \left[ 1 - e^{-i \bar{\omega}_D t} \right] \right\}. \] (10)

Secular-drift-induced phase-mixing
From this slide, we present an explicit calculation of the 3D field induced phase-mixing of ZFs. We consider a single toroidal mode number RMP field as a specific example.

To begin with, we apply a perturbative treatment of the orbit average based on the Lagrangian variation of the magnetic field strength. [J.K. Park et al., PoP ‘09] Instead of direct calculation of the magnetic field strength variation along a convoluted magnetic field line $b$, we consider the magnetic surface deformation perturbatively on the position of an unperturbed field line as follows.

$$B(x) = |B_0(x) + \delta B(x)| \approx |B_0(x_0)| + \delta_L B(x_0),$$

(11)

where $\delta_L B$ is the Lagrangian variation of the magnetic field strength and is given by

$$\delta_L B(x_0) = b_0(x_0) \cdot \delta B(x_0) + \xi \cdot \nabla B_0(x_0).$$

(12)

Here $x = x_0 + \xi$, where $x_0$ is the position on an unperturbed magnetic field and $\xi$ is the displacement of the magnetic field due to the 3D field.

We use following general expression for the magnetic field strength.

$$B = B_0 \left[ 1 - \epsilon \cos \theta + \sum_m \delta_m \cos[(m - nq) \theta - n\alpha + \chi_m] \right],$$

(13)

i.e., $\delta_L B = B_0 \sum_m \delta_m \cos[(m - nq) \theta - n\alpha + \chi_m]$. Here $\alpha = \zeta - q\theta$ is the field line label. We use an ordering $m \sim nq \sim O(1)$ for the RMP field, as referred to before. Note that $(r, \theta, \zeta)$ and thus $\alpha$ are magnetic coordinates defined on the basis of the unperturbed magnetic surface.
3D field induced phase-mixing of residual ZFs

- Note that the general expression of $\bar{\omega}_D$ is proportional to

$$\sum_m \delta_m \sin(n\alpha + \chi_m) \int_0^{\theta_t \text{ or } \pi} d\theta \frac{\cos[(m - nq)\theta]}{\sqrt{\kappa^2 - \sin^2(\theta/2)}},$$

where the pitch angle parameter $\kappa$ is defined as $\kappa^2 = [E - \mu B_0 (1 - \epsilon)]/2\epsilon \mu B_0$.

- We calculate the single-$n$ RMP induced phase-mixing rate using deeply-trapped and strongly-passing approximation near a rational surface.

Following [P.J. Catto and K.T. Tsang, PF '78],

$$\bar{\omega}_D \approx -nq \frac{k_T \rho_T}{R} \sqrt{\frac{E}{T}} \sqrt{\frac{\mu B_0}{M}} \frac{\delta_t}{\epsilon} \sin(n\alpha + \chi_t) \quad \text{for trapped particles}$$

$$\bar{\omega}_D \approx -nq \frac{k_T \rho_T}{R} \sqrt{\frac{E}{T}} \sqrt{\frac{\mu B_0}{M}} \frac{\delta_{m_0}}{\epsilon} \sin(n\alpha + \chi_{m_0}) \quad \text{for passing particles}$$

where

$$\delta_t = [(\Sigma_m \delta_m \cos \chi_m)^2 + (\Sigma_m \delta_m \sin \chi_m)^2]^{1/2},$$

$$\chi_t = \tan^{-1}[(\Sigma_m \delta_m \cos \chi_m/\Sigma_m \delta_m \sin \chi_m],$$

and $m_0$ is the resonant component of the perturbed 3D field.

- Note that the velocity space phase-mixing occurs due to energy dependence of $\bar{\omega}_D$. 
We calculate ZF level in the intermediate and the short wavelength regime following [Lu Wang and T.S. Hahm, PoP '09]. For the long wavelength regime, we calculate the ZF level without using deeply-trapped and strongly-passing approximation.

Resulting long-time \( t \gg \{\gamma_{0i,e}^{-1}, \gamma_{ti,e}^{-1}\} \) asymptotic formula of ZF response is as follows.

\[
\phi_k(t) \approx \phi_k(0) \begin{cases} 
1 - \Gamma_0 \frac{k_r^2 \rho_{Ti}^2}{(1 + T_i/T_e)\gamma_{0i} t} & \text{Long } k_r \rho_i < Q_i < 1 \\
1 & \text{Intermediate } k_r \rho_i < 1 < Q_i \quad (16) \\
\frac{1}{\Gamma(1/4)} \frac{1}{q k_r \rho_{Ti}^2} \frac{1}{(1 + T_i/T_e)\gamma_{0i} t} & \frac{1}{P_{-1/2}(1/\gamma_{0i} t)} \quad \text{Short } 1 < k_r \rho_i < Q_i
\end{cases}
\]

Here \( \gamma_{0i} = n q k_r \rho_{Ti} (v_{Ti}/R) (\delta_{m_0}/\epsilon) \) is a characteristic frequency of the 3D field induced phase mixing (and resulting decay) of residual ZFs.

Note that resonant component \( (m_0, n) \) of the perturbed 3D field plays an important role in the long-time ZF evolution.

We can recover Rosenbluth-Hinton residual ZF level by taking short-time limit \( t \ll \{\gamma_{0i,e}^{-1}, \gamma_{ti,e}^{-1}\} \) of the exact solution of long-wavelength ZF response that we obtained.
Collisionless Decay of Zonal Flows

1. Axisymmetric Tokamak
2. Stellarators
3. Tokamak with 3D RMP Field

Rosenbluth-Hinton

Our Work

Sugama-Watanabe

Both toroidally and helically trapped particles

Lower than RH due to helical trapping

Phase mixing
**RMP induced decay of residual ZFs**

- Figure below is the RMP induced ZF decay for three ZF wavelength regimes plotted for present day tokamak- and RMP-like parameters and representative ZF wavelengths $k_r \rho_T = 0.1$ (Long), $k_r \rho_T = 0.5$ (Inter), $k_r \rho_T = 2.5$ (Short).

In the figure $\gamma = nq(\nu_T/R)(\delta/\epsilon)$.
Zonal Flow Spectra: ITG turbulence vs TEM turbulence

\[ S_{ZF} \]

from T.S.Hahm et al.
GTC simulation of ITG

from J.M.Kwon et al.
GKPSP simulation of TEM
3D field induced phase-mixing vs Collisional damping

- To examine the influence of the 3D field induced ZF phase-mixing, we compare it to the collisional ZF damping rate.
  - Characteristic frequencies
    3D field induced phase-mixing: \( \gamma_{3D} = k_r \rho_{Ti} (v_{Ti}/R)(\delta/\epsilon)nq \)
    Collisional damping: \( \gamma_c = v_{ii}/1.5\epsilon \) [F.L. Hinton and M.N. Rosenbluth, PPCF ‘99]
  - Assumptions for the 3D field: \( \delta \sim 10^{-3}, n = 1 \)
    [J.K. Park et al., PoP ‘09], [Y.M. Jeon et al., PRL ‘12]
  - Parameters of KSTAR L-mode plasma (just before the H-mode transition) near the position of H-mode pedestal top [Y.M. Jeon et al., PRL ‘12], [W.H. Ko et al., NF ‘15]:
    \( R = 2.25m, q = 6, n_e = 0.5 \times 10^{19} m^{-3}, T_i = 0.5keV \quad \Rightarrow \quad \gamma_{3D}/\gamma_c \sim 2.5k_r \rho_{Ti} \)
  - The 3D field induced ZF phase-mixing rate is expected to be comparable to the collisional ZF damping rate for intermediate and short-wavelength ZFs.

Note that fine scale ZFs appear in experiments (H-mode edge [J.C. Hillesheim et al. PRL ‘16]) and nonlinear gyrokinetic simulations of TEM turbulence.
[Y. Xiao and Z. Lin, PRL ‘09], [J.M. Kwon et al., Comp. Phys. Comm. ‘17]

- Since \( \gamma_{3D} \propto T_i \) and \( \gamma_c \propto T_i^{-3/2} \), the RMP induced ZF decay is expected to play a more important role in future tokamaks such as ITER.
Recent experimental results in KSTAR show that the increase of the H-mode transition power threshold is more influential for $n = 2$ RMP than for $n = 1$ RMP. [W.H. Ko et al., KSTAR Conf. O4A-5 (2017)]

Since the RMP induced ZF decay rate is proportional to the toroidal mode number of the RMP field $n$, i.e., $\gamma_{3D} = k_r \rho_{Ti} (v_{Ti}/R) (\delta/\epsilon) n q$, our result indicates more significant increase of the power threshold for higher-$n$ RMP. This $n$-dependence agrees with the trend observed in KSTAR.
Summary

• We extend the previous works of Rosenbluth-Hinton and Sugama-Watanabe to study influence of the externally imposed 3D magnetic field on ZFs in tokamak plasmas.

• Secular radial drift of bounce-centers due to the parallel component of 3D field causes phase-mixing of ZFs, which leads to a long term collisionless ZF decay toward zero level.

• The characteristic frequency of the phase-mixing $\gamma_{0i} = nqk_r\rho_{T_i}(v_{Ti}/R)(\delta_{m_0}/\epsilon)$ is proportional to the toroidal mode number of the 3D field $n$, the ZF wavelength $k_r$, the ion temperature $T_i$, and the (normalized) amplitude of the resonant component of the perturbed 3D field $\delta_{m_0}$.

• The 3D field induced phase-mixing is expected to be comparable to the collisional ZF damping for intermediate and short wavelength ZFs in the present day tokamaks, and to dominate in future tokamaks such as ITER.