Cascade model of multispecies trapped particles turbulence

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Starting point

Gyro-bounce averaged, delta-f equation in $k$-space:

$$
\frac{\partial_t f_{s,k,\ell}}{f_{s,k,\ell}} = i k_{s} j_{0s,k,\ell} \phi_{k} d_{s} F_{0s,\ell} - i k_{s} \frac{E \Omega_{d}}{Z_{s}} f_{s,k,\ell}
\sum_{p+q+k=0} [p_{s} q_{p} - p_{s} q_{p}] J_{0,s,p,\ell} \phi_{p}\star f_{s,q,\ell} - D_{k} f_{s,k,\ell},
$$

Sources:

- injection due to background gradients $\propto d_{s} F_{0s}$
- advection with precession freq. $\propto \Omega_{d}$

Quasi-neutrality:

$$
C_{k} \phi_{k} = \sum_{s} q_{s} \int_{0}^{+\infty} J_{0,s,k,\ell} f_{s,k,\ell} \sqrt{E} dE.
$$

NB: $C_{k}$ contains the adiabatic response of passing particles, polarization effect...
Conserved quantities

Nonlinear term comes from Poisson Bracket:
\[\{\mathcal{J}_{0,s} \cdot \Phi ; \delta f_{s,\ell} \}_{\psi,\alpha} = -\{\delta f_{s,\ell} ; \mathcal{J}_{0,s} \cdot \Phi \}_{\psi,\alpha}\]

Squared distribution function \(\simeq\) entropy:
\[\mathcal{E}_{f_s}^k \equiv T_{0s} \int_0^{\infty} \frac{|f_{s,k,\ell}|^2}{2F_{0s}} \sqrt{E} dE\]
\[\partial_t \mathcal{E}_{f_s}^k = \mathcal{P}_{f_s}^k + \mathcal{N}_{f_s}^k - \mathcal{D}_{f_s}^k, \quad \mathcal{P}_{f_s}^k \propto \partial_\psi F_{0s}\]

Squared potential \(\simeq\) electrostatic potential energy:
\[\mathcal{E}_\phi^k \equiv C_k \frac{|\phi_k|^2}{2}\]
\[\partial_t \mathcal{E}_\phi = \mathcal{P}_\phi^k + \mathcal{N}_\phi^k - \mathcal{D}_{f_s}^k, \quad \mathcal{P}_\phi^k \propto \Omega_d\]

Where, of course:
\[\sum_k \mathcal{N}_{f_s}^k = \sum_k \mathcal{N}_{\phi}^k = 0.\]
Local interactions

Logarithmic, isotropic, discretization:

\[ k_n = k_0 g^n = k_\psi = k_\alpha \]

Keep only nearest neighbours:

\[ \{(n - 2; n - 1); (n - 1; n + 1); (n + 1; n + 2)\} \]

Sabra truncation with gyro-bounce averaged Vlasov equation:

\[
\partial_t f_{s,n,\ell} = i k_n J_{0s,n,\ell} \phi_n d_\psi F_{0s,\ell} - ik_n \frac{E \Omega_d}{Z_s} f_{s,n,\ell} - D_n f_{s,n,\ell} \\
+ \alpha k_n^2 g^{-3} [J_{0s,n-2,\ell} \phi_{n-2} f_{s,n-1,\ell} - J_{0s,n-1,\ell} \phi_{n-1} f_{s,n-2,\ell}] \\
+ \alpha k_n^2 g^{-1} [J_{0s,n-1,\ell} \phi_{n-1}^* f_{s,n+1,\ell} - J_{0s,n,\ell} \phi_{n+1} f_{s,n-1,\ell}] \\
+ \alpha k_n^2 g [J_{0s,n+1,\ell} \phi_{n+1} f_{s,n+2,\ell} - J_{0s,n+2,\ell} \phi_{n+2} f_{s,n+1,\ell}] ,
\]

Quasi-neutrality:

\[
C_n \phi_n = \sum_s q_s \int_0^{+\infty} J_{0s,n,\ell} f_{s,n,\ell} \sqrt{E} \, dE .
\]
$\tau$ scan: linear physics

$\tau = 0.2$

$\tau = 0.713$

$\tau = 2.0$

TIM *linearly* dominant

TIM + TEM with $\approx$ same growth rate

TEM *linearly* dominant
**τ** scan: free energy *spectra*

\[ \tau = 0.2 \]

\[ \tau = 2.0 \]

\[ \tau = 0.713 \]

\[ \star \quad \tau = 0.2 \]

\[ \text{TIM} : \mathcal{E}_\phi \gg \mathcal{E}_{f_i} \gg \mathcal{E}_{f_e} \]

\[ \star \quad \tau = 0.713 \]

\[ \text{TIM + TEM} : \mathcal{E} \gg \mathcal{E}_{f_i} \geq \mathcal{E}_{f_e} \]

\[ \star \quad \tau = 2.0 \]

\[ \text{TEM} : \mathcal{E}_\phi \gg \mathcal{E}_{f_i} \approx \mathcal{E}_{f_e} \]
τ scan: free energy spectra

⋆ $\mathcal{E}_\phi$: no clear exponent max for TIM + TEM
⋆ $\mathcal{E}_{f_i}$: almost identical
⋆ $\mathcal{E}_{f_e}$: same slopes increase with $\tau$
τ scan: productions and dissipations

- $P_{TEM+TIM} > P_{TEM} > P_{TIM}$
- dissipations follow, with large scales $\mathcal{D}_L >$ small scales $\mathcal{D}_s$
Discussion

Summary:

- Not surprising: $\mathcal{E}_{f_e}$ scales with $\tau$
- Not so much surprising: production max for TIM + TEM
- Surprising: $\mathcal{E}_\phi$ systematically higher
- Surprising: $\mathcal{E}_{f_i}$ not changing with $\tau$

TO DO:

- Separate $\mathcal{P}$ into particles $\Gamma_{n_s}$ and heat $Q_{T_s}$ fluxes
- effect of adiabaticity (same spirit as Shaokang’s presentation)
- compare exponents with theory